Material Flow Analysis
and Life Cycle Assessment

TECHNISCHE
UNIVERSITÄT
DARMSTADT

6. Unit: LCA – Life Cycle Inventory (LCI) II

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IWAR

6th Unit: Life Cycle Inventory Analysis (I)

6.1 Calculation of the Product System

6.1 Fundamentals

6.2 Calculation of large Product Systems

Calculation of the Product System

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- For each module, a suitable process flow (reference flow) must be determined.
- The quantitative input and output data of the process module must be calculated in relation to this flow.
- According to the flow chart and process flows between the modules, all flows of all process modules are related to the reference flow.
- The calculation should induce that all input and output data of the system are related to the functional unit.

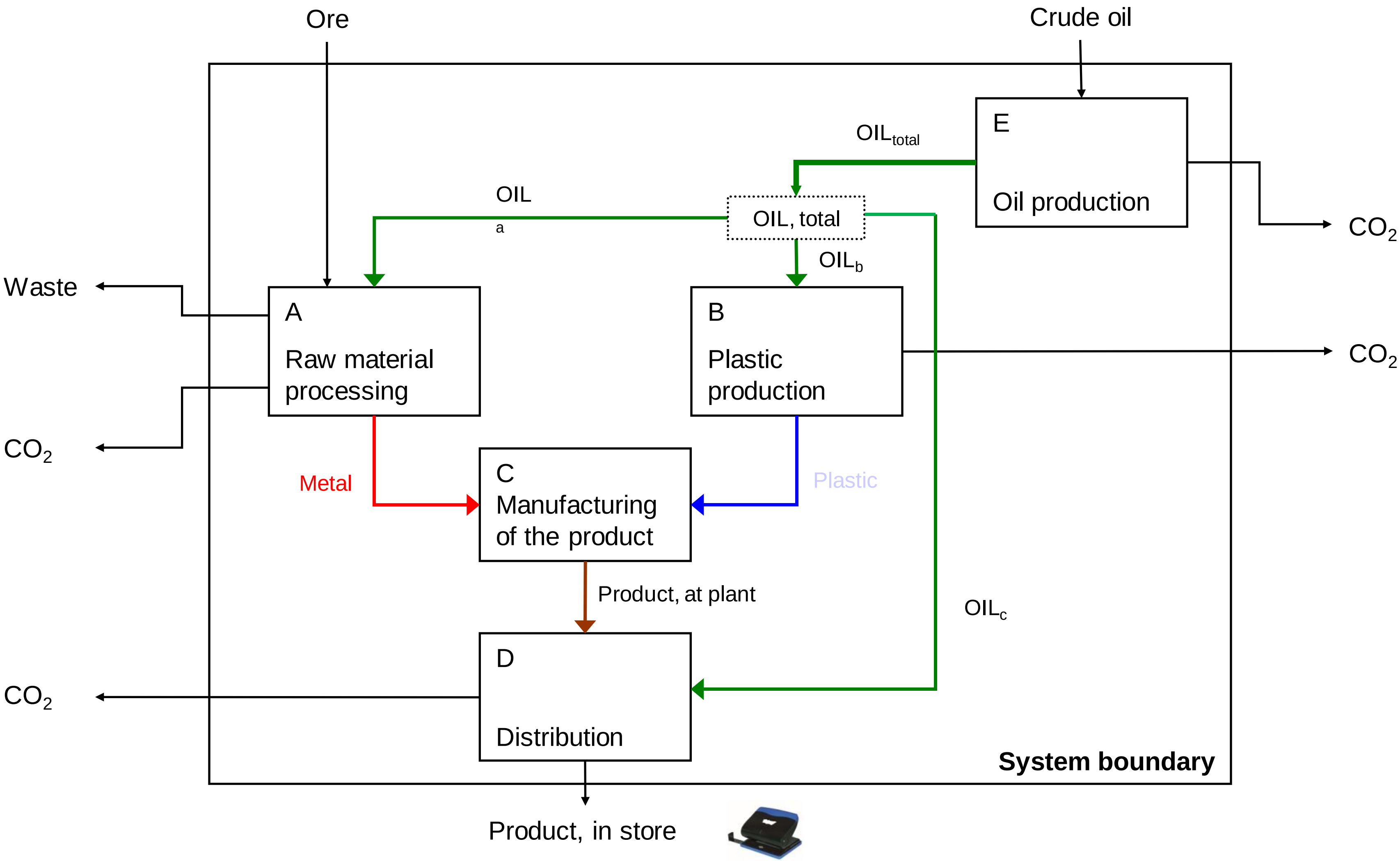
EN ISO 14040:2006 D/E

Calculation - Processing

1. Identify the relevant product of the process ("raw data") and relate the inputs and outputs to a unit of the respective product ("normalisation")
2. Calculation of the (intermediate) product flows (“internal flows") and raw materials related to the functional unit of the system
3. Calculation of the emissions related to the functional unit of the system
4. Addition of all flows of the same elementary flow for the entire system
5. Documentation of the calculation

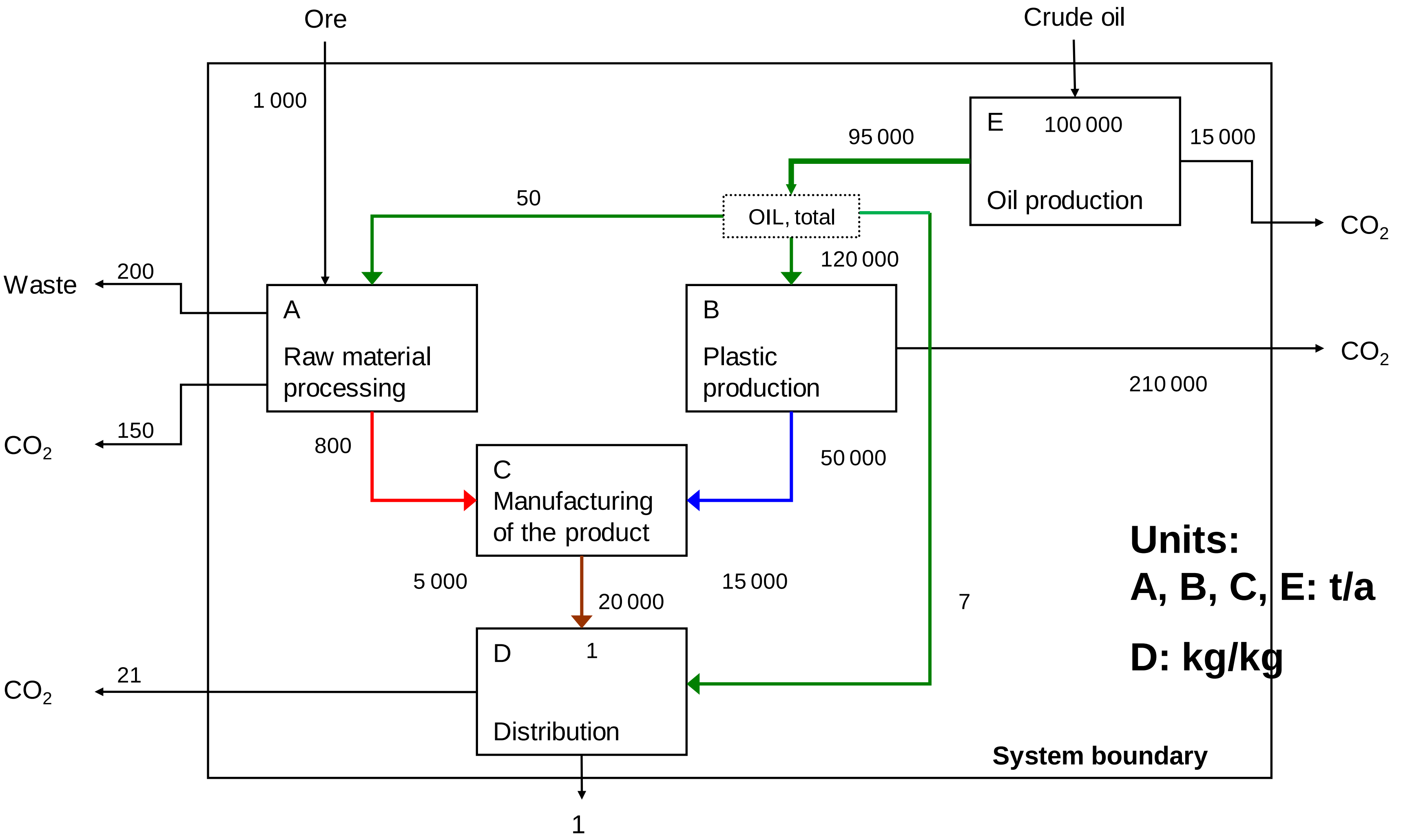
Example: Office Punch

Flow Chart

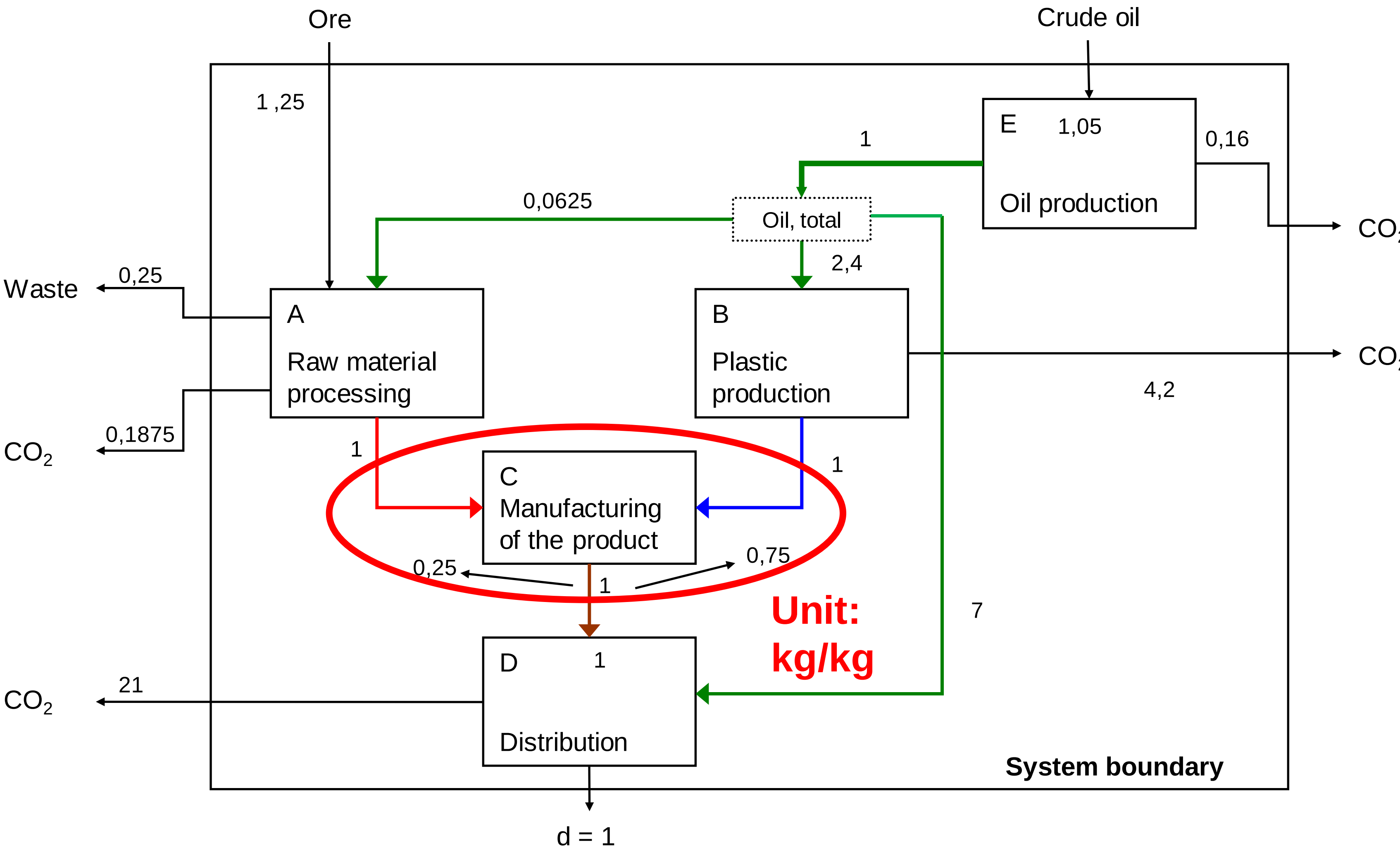


Ore
Crude oil
E
Oil production
OIL_total
OIL
a
OIL, total
OIL_b
CO2
Waste
A
Raw material processing
B
Plastic production
CO2
CO2
Metal
C
Manufacturing of the product
Plastic
Product, at plant
OIL_c
D
Distribution
CO2
System boundary
Product, in store

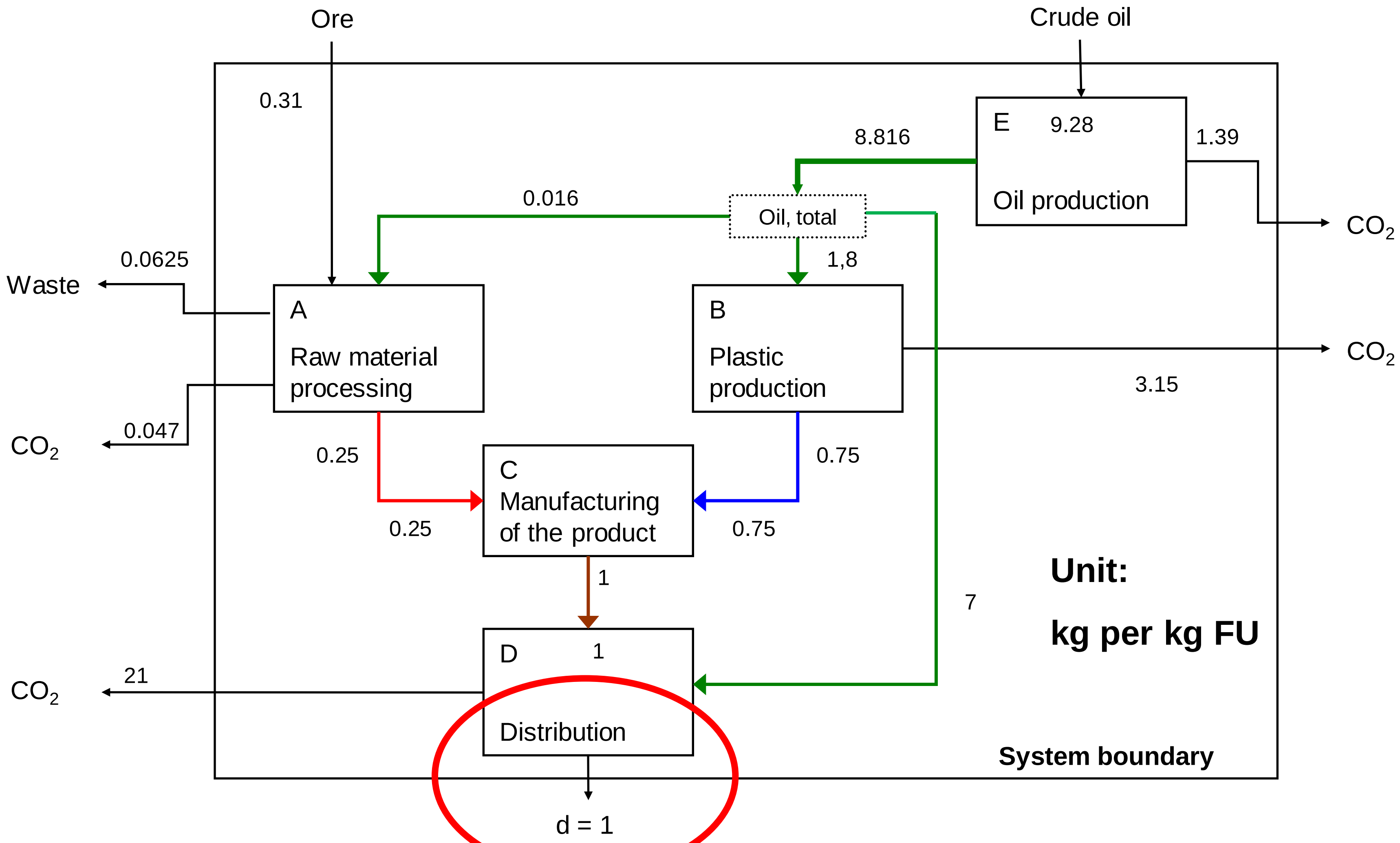
Flow Chart with Raw Data



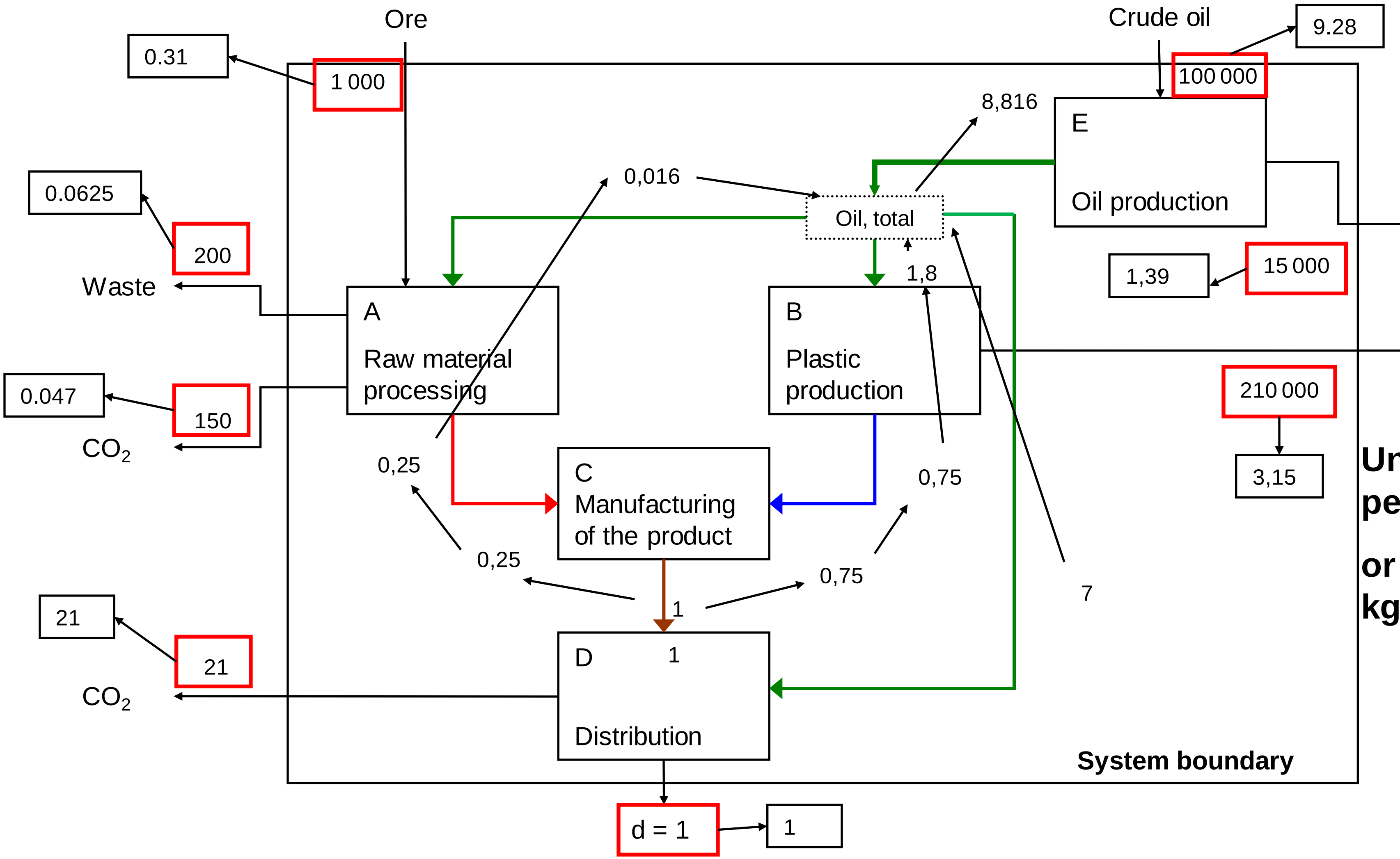
Normalization to a Unit of a Product of the Particular Module



Flows Normalized to the Functional Unit (FU)



Calculation of the Elementary Flows



Unit: Tonne per year

or kg per kg FU

Calculation – Mathematical Representation

1. Identify the relevant product of the process ("raw data") and relate the inputs and outputs to a unit of the respective product ("normalisation")

2. Calculation of the (intermediate) product flows (“internal flows") and raw materials related to the functional unit of the system

3. Calculation of the emissions related to the functional unit of the system

4. Addition of all flows of the same elementary flow for the entire system

5. Documentation of the calculation

6th Unit: Life Cycle Inventory Analysis (I)

6.1 Calculation of the Product System

6.1 Fundamentals

6.2 Calculation of large Product Systems

Problems in Calculating Large Product Systems

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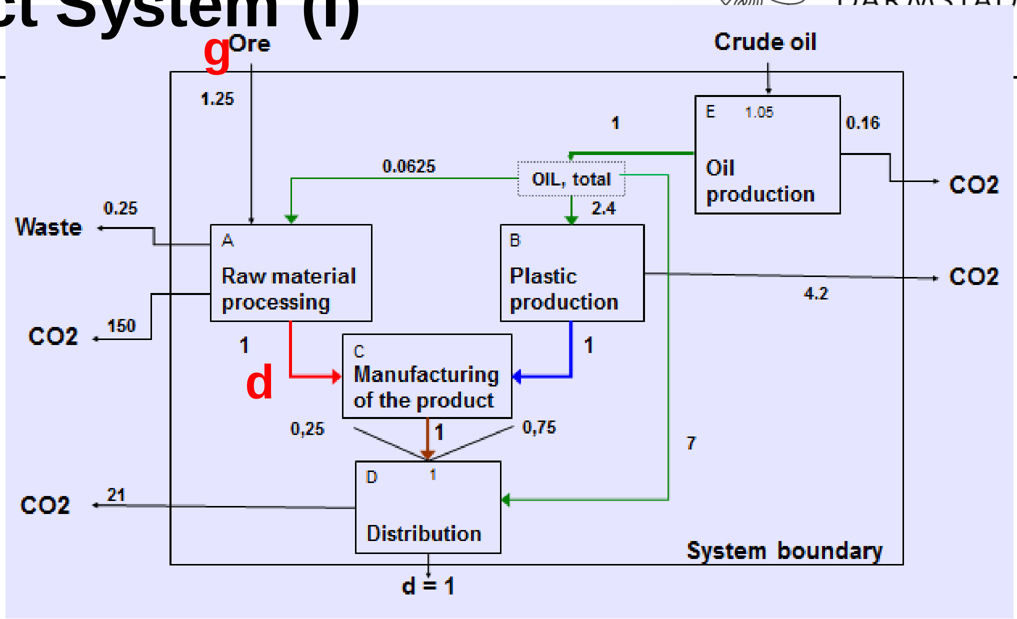
„Large“ product systems:

- Complex products (e.g. car)
- Product systems with high level of detail and capture of many supply chains
- High number of modules, possibly several hundred processes

Problems:

- Calculation "by hand" to complex and error-prone for large product systems .
- Large product system often involve recursions (e.g. recycling loops); approximation procedures and cut-off criteria are necessary when calculating these product systems with sequential calculations.

Interpretation of the Equations for the Calculation of the Product System (I)



a = 1

b = 1 x a

c = 0.75 x b

d = 0.25 x b

e = 0.0625 x d + 2.4 x c + 7 x

f = 1.05 x e

g = 1.25 x d

Output (product) **of the process raw material processing**

Input (demanded quantity of precursor) **of the process raw material processing**

Interpretation of the Equations for the Calculation of the Product System (II)

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$$g = 1.25 \times d$$

Output (product) **of the process raw material processing**

Technical coefficient, characteristic of the production process

Input (demanded quantity of precursor) **of the process raw material processing**

For g = 1.25 [kg ore] und d = 1 [kg metal]:

Technical coefficient $K_{ore;\ metal}$ = 1.25 [kg ore/kg metal]

General Definition of Technical Coefficients

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General:

Definition of input-(technology)-coefficients $a_{i,j}$

Input coefficient = Input („precursor ") of process i per output of process j

$$a_{i,j} = z_j / x_j$$

Calculation of Product Flows of Extensive Product Systems Using Matrix Calculation

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Definitions:

Demand vector f:

- describes the functional unit (FU)
- has n rows
  (n = number of processes or the associated product flows)
- has a 0 in all rows except the row of the FU

Demand matrix (technosphere matrix) A

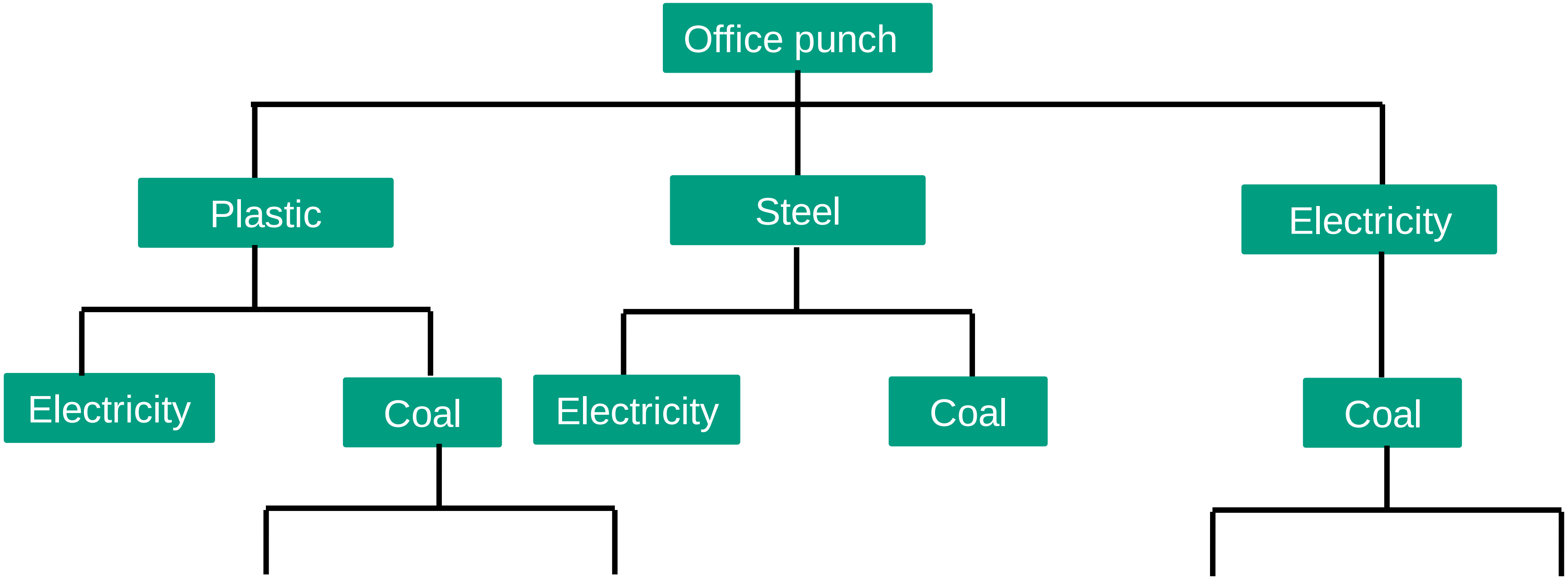
- covers the input coefficients $a_{i,j}$ of all processes of the system
- is a n x n square matrix

Output vector x:

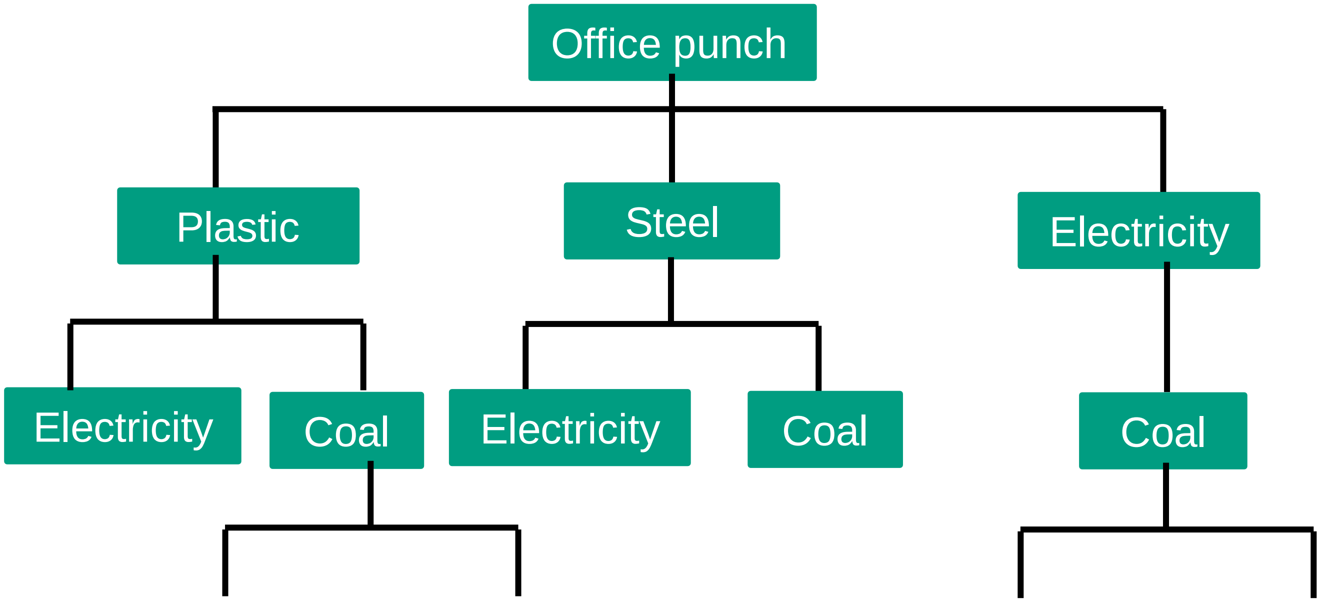
- represents all process activities ("internal flows") that are necessary to provide the functional unit

Calculation of Product Flows of Extensive Product Systems Using Matrix Calculation

Example: product system of the manufacturing of an office punch



Example Office Punch: Identification of the Product Flows



Product flows:

Office punch

Plastic

Steel

Electricity

Crude oil

Example Office Punch: Demand Vector f

f =	Office punch	1
	Plastic	0
	Steel	0
	Electricity	0
	Coal	0

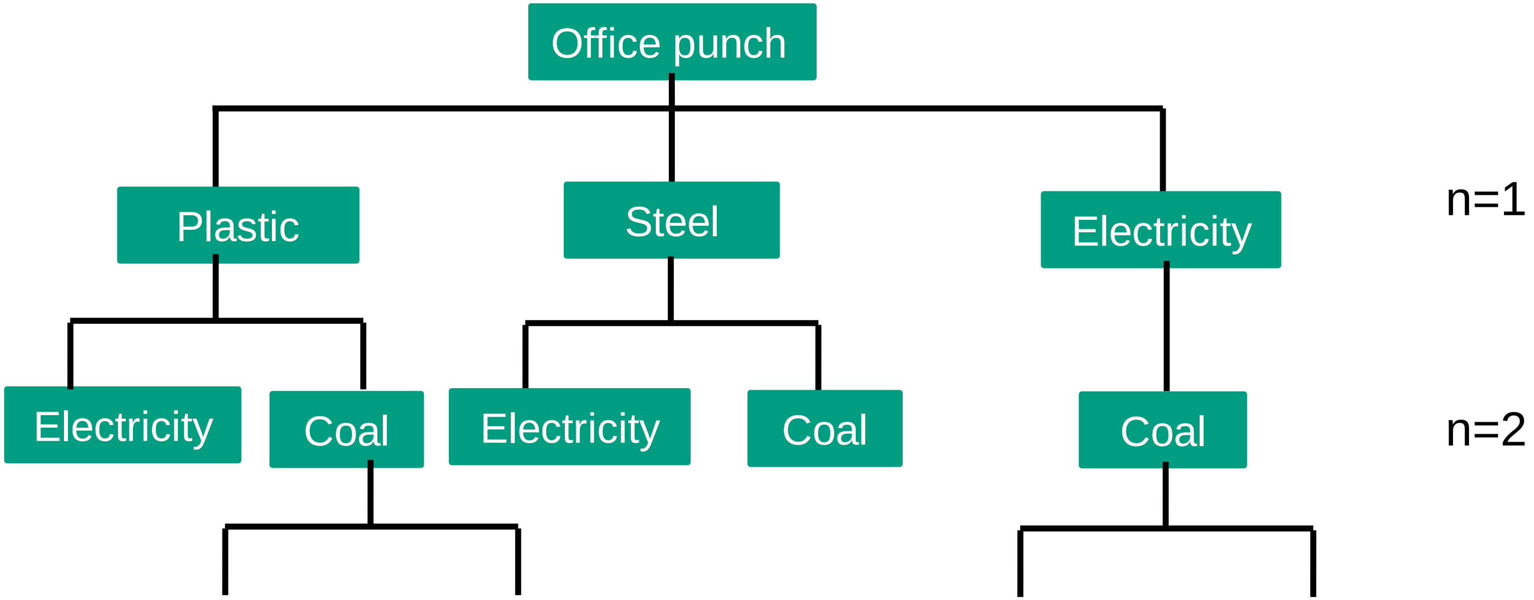
Example Office Punch: Demand Matrix A

	Office punch [quantity]	Plastic [kg]	Steel [kg]	Electricity [kWh]	Coal [kg]
Office punch [quantity]	0	0	0	0	0
Plastic [kg]	0.2	0	0	0	0
Steel [kg]	0.01	0	0	0	0
Electricity [kWh]	0.3	0.2	0.6	0	0
Coal [kg]	0	1	1	5	0

Interpretation of the Demand Matrix A

The demand matrix A covers the activity coefficients and thus represents the **direct references** of all processes, i.e. what amount of inputs does the process get from its direct upstream processes.

Considering the product system as a process tree and regarding the functional unit, this is equivalent to the inputs of stage 1.

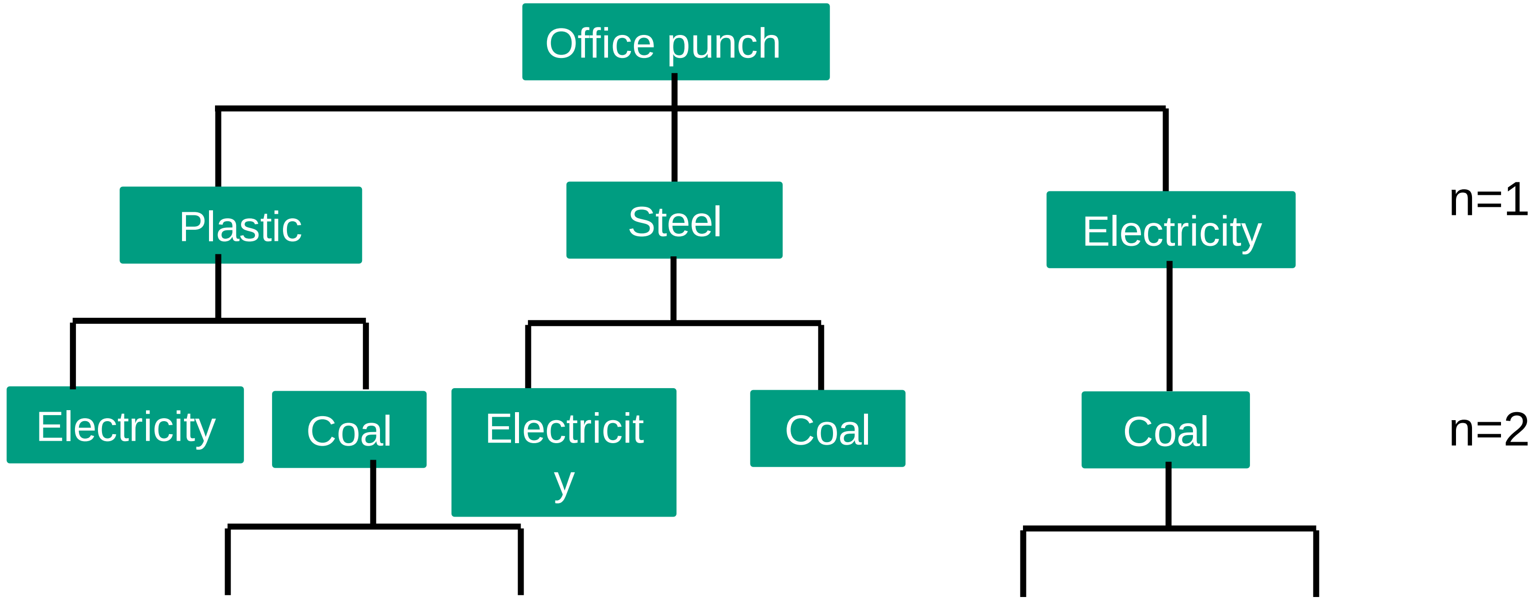


Calculation of the inputs

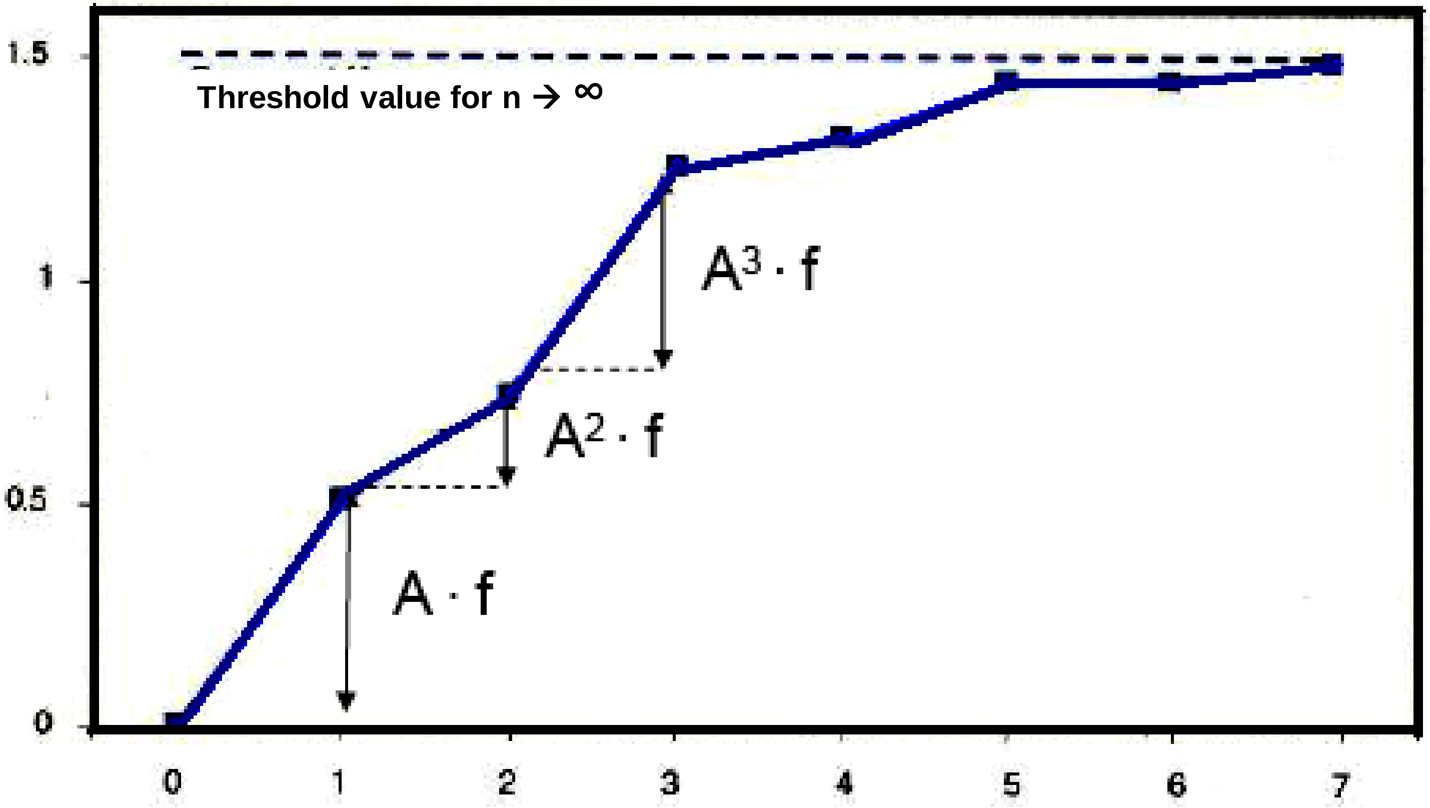
The inputs of stage 2 are obtained by squaring the demand matrix A.

General:

The inputs of stage n are obtained with the n-th power of the demand matrix A.



Calculation of all Inputs



Stage of the process tree (= n)

Calculation of the Output Vector x

$$x = (I + A + A^2 + ... + A^n) \cdot f$$

For $n \to \infty$ applies:

$$= (I - A)^{-1} \cdot f$$

Interpretation:
Calculation of all internal flows (i.e. of the output vector x):

$$x = (I - A)^{-1} \cdot f$$

with I = unit matrix

Example Office punch: Output vector x

Demand matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0 & 0 \\ 0.01 & 0 & 0 & 0 & 0 \\ 0.3 & 0.2 & 0.6 & 0 & 0 \\ 0 & 1 & 1 & 5 & 0 \end{bmatrix}$$

Unit matrix minus demand matrix

$$I - A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -0.2 & 1 & 0 & 0 & 0 \\ -0.01 & 0 & 1 & 0 & 0 \\ -0.3 & -0.2 & -0.6 & 1 & 0 \\ 0 & -1 & -1 & -5 & 1 \end{bmatrix}$$

Inverted matrix $A^{-1} = \frac{1}{|A|}\,(adj\ A)$

$$(I - A)^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 & 0 \\ 0.1 & 0 & 1 & 0 & 0 \\ 0.4 & 0.2 & 0.6 & 1 & 0 \\ 2.3 & 2 & 4 & 5 & 1 \end{bmatrix}$$

Example Office punch: Output vector x

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$$x = (I - A)^{-1} \cdot f$$

For one piece of office punch, one need:

$$x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0.2 & 1 & 0 & 0 & 0 \\ 0.1 & 0 & 1 & 0 & 0 \\ 0.4 & 0.2 & 0.6 & 1 & 0 \\ 2.3 & 2 & 4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.2 \\ 0.1 \\ 0.4 \\ 2.3 \end{bmatrix}$$

Office punch
Plastic
Steel
Electricity
Coal

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Calculation of Emissions of Extensive Product Systems Using Matrix Calculation

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Definitions:

Environmental impact matrix B:

- Covers the emissions per product unit of a process
- Is a m x n matrix
  (m = quantity of the emissions;
  n = quantity of the processes respectively the related product flows)

Environmental impact vector y:

- Is a vector with m rows
- Covers the cumulative emissions of all processes related to the functional unit

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Calculation of Emissions of Extensive Product Systems Using Matrix Calculation

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- By multiplying the environmental impact matrix **B** with the output vector x, the cumulative emissions of all processes are described for each (defined) damage in the environmental impact vector y.

$$y = B \cdot (I - A)^{-1} f = Bx$$

Example Office Punch: Environmental Impact Matrix B

	Office Punch [quantity]	Plastic [kg]	Steel [kg]	Electricity [kWh]	Coal [kg]
CO_2 [kg]	0	2.6	0.53	0.03	0.03
CH_4 [g]	0	5	1	0	0
NO_X [g]	0	4	0.1	1	1

Example Office Punch: Calculation of the Environmental Impact Vector y

$$y = B \cdot (I - A)^{-1} f = Bx$$

$$x = \begin{bmatrix} 1 \\ 0.2 \\ 0.1 \\ 0.4 \\ 2.3 \end{bmatrix}$$

B

	Office punch [quantity]	Plastic [kg]	Steel [kg]	Electricity [kWh]	Coal [kg]
CO_2 [kg]	0	2.6	0.53	0.03	0.03
CH_4 [g]	0	5	1	0	0
NOx [g]	0	4	0.1	1	1

$$y = \begin{bmatrix} 0.6 \\ 0.4 \\ 2.3 \end{bmatrix}$$